UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 4TH FEBRUARY 2016

Organised by the **United Kingdom Mathematics Trust**
from the School of Mathematics, University of Leeds

SOLUTIONS LEAFLET

This solutions leaflet for the IMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

For reasons of space, these solutions are necessarily brief. Extended solutions, and some exercises for further investigation, can be found at:

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1. **B** $6116 - 2016 = 4100$, so $6102 - 2016 = 4100 - 14 = 4086$.

2. **D** The difference between the given options and 1 is $\frac{1}{8}$, $\frac{1}{7}$, $\frac{1}{10}$, $\frac{1}{11}$ and $\frac{1}{10}$ respectively. As $\frac{1}{11}$ is the smallest of these fractions, $\frac{10}{11}$ is closest to 1.

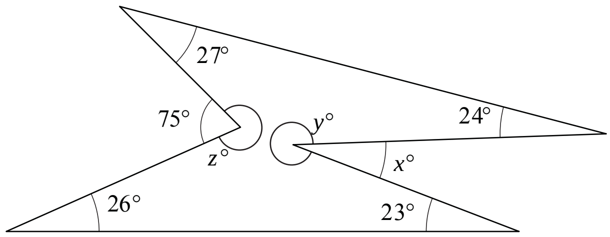
3. **B** The values of the five expressions are 5, 13, 25, 41 and 61 respectively. Of these, only 25 is non-prime.

4. E Amrita bakes every 5 days and Thursdays come every 7 days. So the next time Amrita bakes on a Thursday will be in 35 days time since 35 is the lowest common multiple of 5 and 7.

5. A By train, the distance in miles of the second sign from Edinburgh is $200 - 3\frac{1}{2}$. This sign is halfway between London and Edinburgh, so the distance in miles between the two cities is $2(200 - 3\frac{1}{2}) = 400 - 7 = 393$.

6. C Let g and s be the number of goats and sheep respectively. Then $s = 2g$ and $12 = s - g = 2g - g = g$. Hence the number of animals is $s + g = 3g = 36$.

7. C The angles at a point sum to 360° so $75 + z = 360$ and $y + x = 360$. Therefore $75 + z + y + x = 720$. The sum of the interior angles of a hexagon is $4 \times 180° = 720°$. Therefore

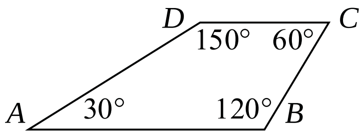


$27 + 24 + y + 23 + 26 + z = 720$, so $75 + z + y + x = 27 + 24 + y + 23 + 26 + z$. Hence $75 + x = 27 + 24 + 23 + 26 = 100$. So $x = 100 - 75 = 25$.

8. E $2.017 \times 2016 - 10.16 \times 201.7 = 201.7 \times 20.16 - 10.16 \times 201.7$
$= 201.7(20.16 - 10.16) = 201.7 \times 10 = 2017$.

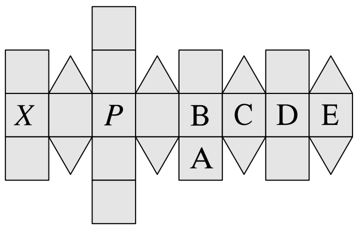
9. C Bertie travelled 5.5 m in 19.6 s, which is just less than one-third of a minute. So his average speed was approximately 16.5 m per minute, which is equal to 990 m in one hour, as $16.5 \times 60 = 990$. Now 990 m = 0.99 km, so Bertie's approximate average speed was 1 km per hour.

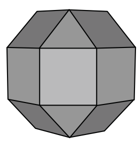
10. E The sum of the interior angles of a quadrilateral is 360°, so $x + 5x + 2x + 4x = 360$, that is $12x = 360$. Therefore $x = 30$ and the angles of the quadrilateral, taken in order, are 30°, 150°, 60° and 120°. The diagram shows the shape of the quadrilateral.



Since $30 + 150 = 180$, we see that AB and DC are parallel. Since it has no equal angles, it is not a rhombus or a parallelogram so it is a trapezium.

11. D When the net is folded up to form the rhombicuboctahedron, the left-hand edge of the square marked X is joined to the right-hand edge of the square marked E so that the eight squares at the centre of the net form a band around the solid. In this band, the square opposite square P is the square which is four squares away from P, that is square D. So if the square marked P is placed face down on a table, then the square marked D will be facing up.





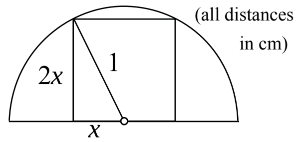
12. D Assume that $a > b$. Then $a + b = 7$ and $a - b = 2$. Adding these two equations together gives $2a = 9$. So $a = \frac{9}{2}$ and hence $b = 7 - \frac{9}{2} = \frac{14-9}{2} = \frac{5}{2}$. Therefore $a \times b = \frac{9}{2} \times \frac{5}{2} = \frac{45}{4} = 11\frac{1}{4}$.

13. B In the seven lines each of the integers from 1 to 7 is used twice and each of the integers from 8 to 14 is used once. So the sum of the numbers in the seven lines is $(1 + 2 + \ldots + 14) + (1 + 2 + \ldots + 7) = 105 + 28 = 133$. Therefore the total of the numbers in each line is $133 \div 7 = 19$.

It is left as an exercise for the reader to show that it is possible to complete the diagram so that the total of the three numbers in each line is indeed 19.

14. D Let there be g girls and b boys in Tegwen's family. Then, as she has the same number of brothers as she does sisters, $b = g - 1$. Also, each of her brothers has 50% more sisters than brothers. Therefore $g = \frac{3}{2}(b - 1)$. So $b + 1 = \frac{3}{2}(b - 1)$ and hence $2b + 2 = 3b - 3$. Rearranging this equation gives $b = 5$. So $g = 5 + 1 = 6$. Therefore there are $5 + 6 = 11$ children in Tegwen's family.

15. A Let the length of the side of the square be $2x$ cm. Then, using Pythagoras' Theorem in the triangle shown, $(2x)^2 + x^2 = 1^2$. So $4x^2 + x^2 = 1$. Therefore $x^2 = \frac{1}{5}$ and the area of the square is $4x^2$ cm^2 $= \frac{4}{5}$ cm^2.



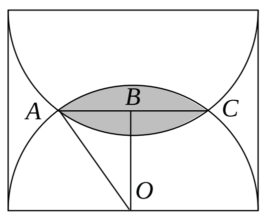
16. B The prime factorisation of $24 = 2^3 \times 3$. Therefore all multiples of 24 must include both 2^3 and 3 in their prime factorisation. Of the options given, only the last includes 2^3. As it is also a multiple of 3, it is a multiple of 24.

17. B Let the radius of the dashed circle be r cm. Then one of the equal areas is bounded by circles of radii of 14 cm and r cm, whilst the other is bounded by circles of radii of r cm and 2 cm. So $\pi \times 14^2 - \pi r^2 = \pi r^2 - \pi \times 2^2$. Dividing throughout by π gives $196 - r^2 = r^2 - 4$. So $2r^2 = 200$, that is $r^2 = 100$. Therefore $r = 10$ (since $r > 0$).

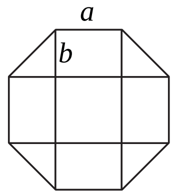
18. C Let the length of each of the shorter sides of the triangle be x cm and the length of its hypotenuse be y cm. Then, by Pythagoras' Theorem: $x^2 + x^2 = y^2$. So $y^2 = 2x^2$. Also, $x^2 + x^2 + y^2 = 72$, so $4x^2 = 72$, that is $x^2 = 18$. Now the area of the triangle, in cm^2, is $\frac{1}{2} \times x \times x = \frac{1}{2}x^2 = 9$.

19. B From the information given, there are at least two 9s in the list, since 9 is the mode, and at least one number greater than 10, since 10 is the mean. So there are at least three numbers greater than 8 in the list. Therefore the list must contain at least six numbers, as the median of the numbers is 8. Moreover, it is possible to find suitable lists of six numbers with sum 60 (as the mean is 10), for example 1, 2, 7, 9, 9, 32.

20. D In the diagram, O is the centre of the lower semicircle, A and C are the points of intersection of the two semicircles and B is the point at the centre of the rectangle and also of the overlap. Now OA is a radius of the semicircle so OA has length 5 cm. Also OB is half the height of the rectangle so has length 4 cm. Angle ABO is a right angle. So triangle ABO is a (3, 4, 5) triangle and hence $AC = 2 \times 3$ cm $= 6$ cm.



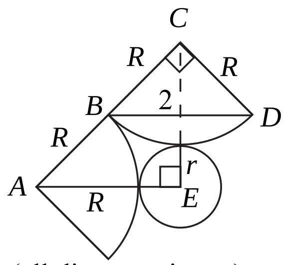
21. A Let a be the side length of the octagon and b be as shown on the diagram. The square in the centre is a by a, each rectangle is a by b and the triangles are each half of a b by b square. Applying Pythagoras' Theorem to a triangle shows that $a^2 = 2b^2$. So the shaded area is $b^2 + ab = b^2 + \sqrt{2}b^2 = b^2(1 + \sqrt{2})$.



Similarly the total area of the figure is $a^2 + 4ab + 2b^2 = 4b^2 + 4\sqrt{2}b^2 = b^2(4 + 4\sqrt{2})$. Therefore the ratio required is $(1 + \sqrt{2}) : (4 + 4\sqrt{2}) = 1 : 4$.

22. E For brevity, let T denote a truth teller and L a liar. Clearly each T has to have an L on each side. Each L either (i) has a T on each side or (ii) has an L on one side and a T on the other side. The largest number of Ts will occur if (i) is always the case. This gives the arrangement TLTLTL… which, since 2 divides 2016, joins up correctly after going round the table. In this case the number of Ts is $\frac{1}{2} \times 2016$. The smallest will occur if case (ii) always is the case. This gives the arrangement LLTLLTLLT… which, since 3 divides 2016, also joins up correctly. In this case the number of Ts is $\frac{1}{3} \times 2016$. The difference is $\frac{1}{6} \times 2016 = 336$.

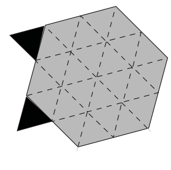
23. B The diagram shows part of the figure, to which have been added A and C, centres of two of the quarter-circle arcs, B and D, points of intersection of two arcs, and E, the centre of the small circle. In cm, the radii of each arc and the small circle are R and r respectively. Firstly, note that $\angle BCD$ is a right angle as arc BD is a quarter of a circle. Therefore, by Pythagoras' Theorem $R^2 + R^2 = 2^2$ so $R = \sqrt{2}$. Consider triangle ACE: from the symmetry of the figure we deduce that $\angle AEC = \frac{1}{4} \times 360° = 90°$. So, by Pythagoras' Theorem $(R + r)^2 + (R + r)^2 = (2R)^2 = 4R^2$. Therefore $(R + r)^2 = 2R^2 = 2 \times 2 = 4$. Hence $R + r = 2$, so $r = 2 - R = 2 - \sqrt{2}$.



(all distances in cm)

24. C Let the distance from the bottom of the escalator to the top be d. Then, when she stands still, Aimee travels $d / 60$ every second. When she is walking, Aimee travels $d / 90$ every second. So when Aimee walks up the working escalator, the distance which she travels every second is $\frac{d}{60} + \frac{d}{90} = \frac{3d + 2d}{180} = \frac{5d}{180} = \frac{d}{36}$. So the required number of seconds is 36.

25. D The tiled area may be considered to be a tessellation of the figure shown, except for the dotted lines. For every hexagonal tile, there are two triangular tiles. The diagram shows that the area of each hexagonal tile is 24 times the area of each triangular tile. As there are two triangular tiles to each hexagonal tile, the ratio of the fraction of the floor shaded black to that which is shaded grey is 2 : 24 = 1: 12. Therefore, in the repeating pattern of tiles, the fraction which is shaded black is 1/13.



The exact ratios given are for the infinite plane. Since we are dealing with a finite floor, this is approximate since the edges are unpredictable, but close to correct since the numbers involved are large.